

## INSIGHTS ON THE RELATIONSHIPS BETWEEN MATHEMATICS KNOWLEDGE FOR TEACHERS AND CURRICULAR MATERIAL

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*This study reports teachers' insights and challenges after one year of adopting a curricular material designed to move students through carefully engineered, small steps and encourage learners through success and accessible challenges. The analysis of interviews showed that teachers 'followed' the material in different ways, not necessary in-line with its underlying principles. Two of these principles—bonusing and breaking down concepts into smaller elements—were particularly difficult for many teachers, suggesting the need of a specific teachers' mathematical knowledge.*

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### Introduction

While there is extensive research on both mathematics teachers' knowledge and the quality of curricular materials, the number of studies combining these two factors is limited. In an effort to address this gap, Charalambous and Hill (2012) reported a multiple case study suggesting that curricular materials can increase quality of instruction if they are supported and followed properly. Understanding the relationships between mathematics teachers' knowledge, curricular materials, and student performance would inform policy decisions regarding adoption and implementation of new resources, as well as the design of corresponding professional learning opportunities for teachers.

This paper analyzes one case of an elementary school adopting new curricular material and engaging teachers in corresponding professional learning over the course of one year. The study was conducted as part of a broad, longitudinal project, the Math Minds Initiative, involving a school district in western Canada, researchers from the University of Calgary, and the JUMP Math (2015) organization. The initiative focused on a particular school with a history of low performance in mathematics. The purpose of the initiative was to improve mathematics teaching and learning at the elementary level and to understand the relationship between curricular resources, teachers' knowledge and students' performance. We are interested in what teachers need to know in order to teach mathematics well, and how this knowledge can be supported through access to particular resources and related teacher professional development. As design-based research, this study draws on multiple sources of data informing next steps in the initiative. However, the focus of this paper is on teachers' experience of adopting the JUMP Math program. Specifically, we address the question: what were the insights and challenges perceived by teachers during the first year that all teachers at the school adopted the JUMP material?

Understanding teachers' insights sheds light on teachers' learning through the year, as well as knowledge required to adopt the JUMP Math materials. Teachers' challenges during this project provide information about the knowledge required not only for the adoption of the material, but also for quality mathematics instruction in general.

### Curricular Material and Mathematics Knowing for Teachers

Teachers' disciplinary knowledge of mathematics has been a focus of research since the 1970s. With an initial emphasis on formal mathematics content, over the last few decades, the main interest has shifted to more varied aspects of mathematics knowing such as access to a diversity of meanings for concepts, beliefs on the nature of the subject matter, and how knowledge is enacted in the classroom (Davis & Renert, 2014; Thompson, 2015). While there are efforts to measure this knowing through tests, such as the instrument proposed by Thompson, we concur with Davis and Renert's argument that such knowing includes an open disposition and cannot therefore be readily measured with tests and other instruments. Two features of this disposition are relevant for this report. First, teachers have to be responsive to students' mathematical conceptions and misconceptions. They should be continuously aware of students' potential interpretations of a concept. Second, school mathematics is not limited to standard definitions, notations and algorithms such as those reflected in a program of studies. Teachers should be open to enact mathematics as a creative, emergent activity, which involves mathematical explorations and inquiry beyond textbooks that may result in insights not only for students, but also for teachers.

Teachers draw from a variety of resources including textbooks, teachers' guides, online material, electronic devices, and the community (Clark-Wilson et al. 2014; Gueudet, Pepin, & Trouche, 2013; Gueudet & Trouche, 2009). Following Gueudet, Pepin and Trouche, we conceive the adoption of curricular material as a creative act: "teachers' work with resources includes selecting, modifying, and creating new resources, in-class and out-of-class" (p. 1003). Gueudet and Trouche proposed the term *documental genesis* for the evolving process of the manner in which teachers use a resource. A *document*, for a particular teacher in given moment, consists of a resource and a scheme of utilization. As the scheme of utilization changes over time, a document is dynamic, whereas the resource may remain unchanged. The process of document genesis is twofold: "The *instrumentalization* dimension conceptualizes the appropriation and reshaping processes ... The *instrumentation* dimension conceptualizes the influence on the teacher's activity of the resources she draws on [emphasis added]" (Gueudet & Trouche 2009, p. 205). Most recently, Gueudet et al. (2013) considered a collective dimension of document genesis including joint work on selecting and adapting educational resources. We extend the idea of document genesis to a more ecological perspective in which the community includes not only other teachers, but also the research team and professional learning facilitators. The teacher participants are *coupled* with the researchers and facilitators in a process of mutual influence (Preciado Babb, Metz, Marcotte, 2015). In this sense, our perception as researchers of curricular material is also influenced by our interactions with teachers and informed by the data collectively gathered and analyzed for research purposes.

### The Math Minds Initiative

The Math Minds Initiative is a five-year project started in 2012. While the school district provided a research school as a main focus for the study, the team from the University of Calgary provided professional support to teachers from this school as well as from other schools in the district. The JUMP Math organization contributed the mathematics program as well as further support for professional learning. During the first year of the initiative, two teachers started mid-year to use the program with no further support. In 2013 all the teachers were required to adopt JUMP Math as official curricular material and to attend the corresponding professional development sessions through the year.

The curricular material provided by JUMP Math consisted of teachers' guides, an assessment and practice book for each student, and access to pre-designed SmartBoard slides. Additionally, students were provided with individual mini-whiteboards—a suggestion from the research team to assist with the continuous assessment recommended by the resource package.

## JUMP Math Principles

The Canadian version of JUMP Math is based on both the *Western-Northern Canadian Protocol for Collaboration in Education* (WNCP, 2006), which provides guidelines for the curriculum in several provinces in Canada, and the Ontario program of studies. The teachers' guide (Mighton, Sabourin, & Klebanov, 2010) provides lesson plans with references to each particular outcome in the corresponding program of studies—WNCP or Ontario. The lesson plans correspond to the assessment and practice book and include individual and group activities and explanations. The guide shows teachers how to introduce one concept at a time, explore concepts and make connections in a variety of ways, assess students quickly, extend learning with extra bonus questions and activities, and develop problem-solving skills. It also provides support material for each strand.

The JUMP Math program is based on a number of principles, including confidence building, guided practice, guided discovery, continuous assessment, rigorously scaffolded instruction, mental math, and deep conceptual understanding. While the assessment and practice book consists of sequences of exercises, the teachers' guide has numerous suggestions for engaging students in discovery and problem solving. The guide also encourages students' independent thought and work: "When you feel your students have sufficient confidence and the necessary basic skills, let them explore more challenging or open problems" (Mighton et al. 2010, p. A-5). The JUMP material shows teachers how to break the material into steps and assess component skills and concepts. It teaches "fundamental rules, algorithms, and procedures of mathematics for mastery, but students are enabled to discover those procedures themselves (as well as being encouraged to develop their own approaches) and are guided to understand the concepts underlying the procedures fully" (p. A-6).

Despite the seemingly direct approach to instruction, every lesson in the teachers' guide refers to at least one problem solving strategy, including: looking for patterns; changing into a known problem; reflecting on other ways to solve a problem; doing a simpler problem first; making and investigating conjectures; using mental math and estimation; representing; guessing, checking and revising; selecting tools and strategies; using logical reasoning; justifying the solution; and revisiting conjectures that were true in one context. An important component of the program is *bonusing*, which involves extensions of concepts and skills in each lesson. The teachers' guide advises teachers to "be ready to write bonus questions on the board from time to time during the lesson for students who finish their quizzes or tasks earlier" (Mighton et al. 2010, p. A-8). Lessons in the teachers' guide include examples of such questions, and teachers are encouraged to create their own. Strategies to create bonus question include: change to larger numbers or introduce new terms or elements; ask students to correct mistakes; ask students to complete missing terms in a sequence; vary the task or the problem slightly; look for applications of the concept; look for patterns and ask students to describe them.

## Method

The Math Minds Initiative is design-based research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) that includes the implementation of the curricular material, as well as professional development aimed at improving mathematics literacy in a school with a long history of low achievement—as well as other schools in the district. The initiative also aims at further research and theory on mathematics teacher knowledge. The research project includes multiple sources of data such as video-recorded lessons, class observations, longitudinal results of the Canadian Test of Basic Skills (CTBS, Nelson 2014), and interviews with teachers and students. In this paper we present the analysis of six semi-structured interviews with teachers who taught during the school year 2013-2014 at the research school. Examples of the interview questions are: What specific advice would you give to new teachers joining Math Minds? Have you found [JUMP Math] materials to be helpful? Restrictive or difficult? To what extent did you follow the teachers' guide? SmartBoard lessons?

Workbook? In what ways did you improvise / extend / elaborate? Have you found [JUMP Math] principles helpful? Restrictive or difficult? What are your goals or priorities for improving your teaching of math?

Transcripts of the interviews were coded using NVivo with a particular focus on the manner teachers used the resources to capture the documentation process. The initial codes were compared each other, forming broader categories. Four major categories resulted from the analysis, which included the codes with higher prevalence. These categories are consistent with class observations conducted by different members of the research team.

## Results

We present the results in four sections, corresponding to each major category. While the first category refers to how teachers used the material in general, the other three are more specific to JUMP Math principles. Excerpts from the interviews are included as evidence to support our findings.

### Document Genesis

Teachers claimed that they followed the teachers' guide and used JUMP Math materials consistently at the beginning. Some tried to fully complete all the pages in the assessment and practice books that applied to the official program of studies and to use all of the associated lessons in the teachers' guide and, often, all of the associated SmartBoard slides. This is evident in the following excerpt:

*Teacher:* Whereas I think when you first begin, you feel like, okay, I've got to go through each one, and it just wasn't working. So again, it's just the experience and sort of knowing, okay—and obviously previewing the slides and saying, okay, we don't—we can skip this one, or this—unless they're really struggling or—and just being able to know where can I stop and how much do I really need to go through all of this.

While it was clear for teachers that some slides or parts of a lesson would have to be selected, the motivation for such decisions varied. The previous excerpt suggests that the teacher made the decisions based on assessment of students' struggles. However, other motivations included both time pressures and a-priori judgments that some steps were not needed, as is evident in the following excerpt:

*Teacher:* Towards the end, when I was trying to catch up a bit, I was taking the teacher guide and I was looking at the outcomes and what our curriculum outcomes were, and if it was like ... number sense ... in four lessons, then I would look at those four lessons, see what the big picture was, because then I could condense them maybe to two lessons instead of four.

A third type of motivation identified in the interviews was familiarity with another, previously used resource. One teacher commented that it was easier to use a resource she was already familiar with, as long as it was similar to what was suggested in the JUMP Math materials:

*Teacher:* Well I have one that's very similar that will still teach the same outcome, but it's a different game in a little bit of a different way.... Taking what I've had from my past as a teacher, because it worked, it was good. Is it the activity in the JUMP lesson? No, but it worked. And so it would save me some time that way, because it does take a lot of time to prep for these, so I would have something like that, maybe use that game instead.

Finally, another manner in which teachers used the material was to select pages from the practice book for bonusing:

*Teacher:* I try to follow [the teachers' guide] exclusively. The SmartBoard lessons, like I say, some of them—if they're very hands-on, I will use a lot of them. ... I just make sure that I've looked through [the material] and then I just pull up those two or three that I need. And the workbook, I look at it: is this going to be for everybody or is it going to be a bonus page?

This last excerpt shows a decision based on two JUMP Math principles: continuous assessment and bonusing.

### Continuous Assessment

All teachers mentioned continuous assessment in the interviews. They also consistently referred to the use of the small whiteboards to assess students in-the-moment. Overall, the material seemed to impact teachers' knowledge regarding this fine-grained presentation of concepts and procedures, as well as the corresponding assessment practice. Continuous assessment not only served to break the content into small pieces, so everybody understands the concept, skill or instruction in class, but also to inform decisions about whether to skip parts that might already be mastered. These decisions, however, seemed to be more difficult to make, as suggested in the following teacher's comment:

*Teacher:* I feel like I need to speed up. I don't know. I need to become better at just moving on and not getting hung up on things and being able to recognize when we can move on and at the same time—and so it's not—at the same time, not compromising that in-depth study of things. Like knowing where, hey, they got it, we can go. We don't need to keep doing this.

Breaking down into smaller steps and constantly assessing students was particularly relevant to a teacher who had been a successful mathematics student:

*Teacher:* I was very successful in math as a student, and I just get it, and I find it difficult to do those microsteps back as to how to make it simpler for the kids and simplify it. And when I taught it that way, I'm like, oh my God, I don't know how to teach it a different way, because I just get it. And so I don't see a different way to get there, and I think that's my biggest challenge because I've never struggled with math. As a student, I was very, very successful, but that makes teaching math harder, because I don't know how to attack a problem from a child's perspective.

This excerpt is consistent with Davis and Renert's (2014) notion of the teacher being an expert who is able to appreciate the struggles of a novice.

### Bonusing

All teachers made reference to bonusing. However, all but one claimed that finding and creating bonus questions and tasks was challenging. Although the teachers' guide shows how to create bonus questions and the assessment and practice book has bonus questions, one teacher perceived the need to find bonus questions beyond the material:

*Teacher:* You need to find bonus questions often from a variety of other sources beyond the JUMP resource in order to find the proper challenge for each individual child.

This comment also highlights a perceived need to personalize bonus questions. The following statement reflects a similar assumption:

*Teacher:* Coming up with really good ones has taken a lot of time, a lot of effort. But I feel now I've got a better idea of kind of what works for the kids as well and also just realizing not every kid's going to have the same bonus question, right? Like you're going to change the bonus question based on the kid and kind of the extra challenges that they need.



In contrast, the teacher who found it easy to create bonus questions claimed:

*Teacher:* Everybody is so engaged in the workbooks and so it gives me an opportunity to continually assess their learning and because there—there is generally enough in the workbook that everybody has enough to do, and it's easy. Having said that, it's very easy to create challenges from—because of the way that the questions are structured, because of the way the work is structured. It's very, very easy to just create challenges on the spot for those who need it. And in a lot of cases, the students will create challenges on their own—their own challenges.

For this teacher, bonus questions and tasks were easy to create on the spot by following the structure of questions in the material. The excerpt also suggests a culture of self-bonusing in her classroom.

### **Inquiry and Problem Solving**

Teachers consistently indicated a lack of opportunity for problem solving or inquiry in the JUMP Math approach. However, most of them indicated that going through the mini-steps was necessary, and that the program did this very well. An example of a teacher's perception on inquiry in the material follows:

*Teacher:* And when I—and as far as inquiry goes, that is our direction in education in the next ten years, and as soon as I heard that, I thought, well, JUMP doesn't lend itself to inquiry. But in thinking about it, it certainly can, it just has to—it's maybe how we're going to start praising things but once again, I still think we—we need the foundation before we can even [missing word?] an inquiry. And so my struggle this year is sometimes should it—should I just do like an inquiry lesson or should I stick with my microsteps, but I want to do the microsteps because I'm learning so much about what I missed teaching them. So to me, right now, that's more important and maybe we throw in an inquiry day on Fridays or something. Throw in everything and just give them an open-ended question and maybe change that next year. It's just this year I'm just sticking to my recipe.

In the previous excerpt, the teacher gave second thought to the possibility of including inquiry in the JUMP Math approach. However, the last comment regarding sticking to the recipe suggests that she did not see inquiry addressed in the material.

There was a particular comment regarding students not being used to more complex, or multistep problems:

*Teacher:* So all of a sudden, when [students] had to do this sort of a—in a way, it was a multistep problem, whereas the vast majority of this program is very one step questions and these microsteps. So as soon as you throw a multistep problem at them, I was very surprised at how many kids were just like, whoa, what am I going—how do I solve this? And there was just no—not even an attempt to work through the problem.

Overall, teachers' perceptions of inquiry and problem solving seem to be contrary to the problem solving strategies included in the teachers' guide.

### **Discussion**

The analysis of teachers' interviews presented in this paper yields several conclusions regarding the interactions of the classroom resources and mathematics knowledge for teaching. First, the analysis of document geneses showed that teachers' interpretation of what it means to follow JUMP

Math were very different. The initial approach of having all students cover all the material contrasts with the approach based on assessing students and selecting pages from the assessment and practice book for bonus. The latter approach seems to be more aligned with the philosophy of the program.

Second, all the teachers made reference to the incremental steps and to continuous assessment. In particular, the use of the mini-whiteboards supported continuous assessment during class. It is particularly interesting that the teachers who claimed having no problem with mathematics when she was a student found it difficult to break concepts into smaller steps. The use of the resources enabled this realization; however, the resource did not seem to enable her to deconstruct concepts appropriately. This suggests that even if teachers know that the resources were designed around microsteps, they may experience difficulty in breaking concepts and skills into smaller elements themselves. This research finding is consistent with a most recent analysis of teachers' perception of scaffolding in the year proceeding the interviews reported in this paper. Sabbaghan, Metz, Preciado Babb, and Davis (in press) found that teachers with less experience in the initiative tended to use traditional strategies for scaffolding—such as modeling and coaching—in contrast to teachers with more than one year in the initiative who considered micro-level scaffolding strategies.

Third, even though the teachers' guide provides advice on bonusing, most teachers found this very challenging. The idea of bonusing has been evolving during the Math Minds initiative. The research team has compiled examples from teachers implementing the program. The team has also identified connections to the literature on intrinsic motivation, shaping the collective understanding of the bonus principle of the JUMP Math program. Moreover, in contrast to the teachers' guide's emphasis on creating bonus for early finishers, Mighton (2007) also advised to consider bonus questions for everyone: "I always make up special bonus questions for the most challenged students, too, so they can feel that they are doing harder work as well" (p. 106). We have come to perceive bonusing as a strategy for both fostering a positive attitude towards mathematics and deepening mathematical understanding.

The longitudinal results for student performance on the CTBS tests—omitted in this paper due to limited space—showed a significant improvement after one year of adopting the JUMP Math program (Metz, Sabbaghan, Preciado Babb, & Davis, in press). This was particularly reflected in students who initially had low performance. However, scores leveled or decreased for some students with initially high performances. Our principal hypothesis for this situation is that it might be attributed to teachers' lack of confidence in creating bonus questions for students. This hypothesis is supported by the fact that the students of the one teacher who reported confidence with bonusing showed significant improvement across the board (Preciado Babb, McInnis, Metz, Sabbaghan, Davis, in press).

Finally, the general agreement that a resource like JUMP Math does not include inquiry contrasts with the problem solving strategies included in each lesson in the teachers' guide. This is probably due to a strong focus on the assessment and practice book instead of the suggested activities in the guide and the SmartBoard slides. The research team considers that both bonusing and the selected sequence of tasks (Metz, et al., in press) in the assessment and practice book afford opportunities for mathematical inquiry. There is, therefore, a need to better understand the mathematical knowledge required for bonusing and for breaking down concepts into smaller elements.

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